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An experimental investigation of stability and interfacial waves in co-current flow of two liquids

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The stability of the co-current stratified flow of oil and water was investigated experimentally in a horizontal rectangular conduit. Laminar-turbulent transitions were determined for both phases. With the two-phase system the transition to turbulence in the water phase occurred at a higher Reynolds number in the presence of a laminar oil layer provided the input water-to-oil ratio was relatively high, while the transition in the oil phase took place at a lower Reynolds number in the presence of a turbulent water layer. The appearance of first interfacial waves coincided with the transition to turbulence of the less viscous or water phase. This suggests that in the system investigated the resonance mechanism as proposed by Phillips (1957) was responsible for the generation of these first waves. However, at relatively high water flow rates and water-to-oil ratios more pronounced waves were observed which appeared to be generated by an instability in the mean flow.

Introduction

An understanding of the mechanism responsible for generating and maintaining wavy disturbances at the interface between two fluids is of importance to many engineering problems. Ursell (1956) has reviewed the work done on the generation of waves by wind action. More recently Phillips (1957), Miles (1957, 1959*a*, *b*, 1962), and Benjamin (1959) have explored new models or modified the ones discussed in Ursell's review for the transfer of energy from wind to surface waves. The effect of turbulent pressure fluctuations in one of the phases was considered by Phillips, who suggested a resonance mechanism for the generation of interfacial waves. On the other hand, Benjamin and Miles proposed a theory for interphase energy transfer based on the variation of pressure and shear stresses over a wavy surface.

As Ursell in his review has pointed out, the problem can be considered as one of mathematics, provided it is accepted that the fluid motions are subject to the laws of classical mechanics with the appropriate boundary conditions. The resources of the mathematics are unfortunately insufficient at present for a complete solution, and significant progress can only be made with certain physical simplifying assumptions. The immediate task lies then in finding the

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appropriate assumptions, and this can only be done through close co-operation of theory and experiment.

Different types of surface disturbances have been found to exist. Hanratty & Engen (1957) have classified the various interfacial structures observed in a laboratory channel with stratified air-water flow as:

(1) smooth,
(2) two-dimensional waves,
(3) three-dimensional waves,
(4) roll waves.

Attempts to predict the appearance of these waves in the gas-liquid system based on the Miles–Benjamin theory have met with considerable success (Cohen 1964; Hanratty & Hershman 1961). It is not known, however, if similar theories are applicable to other systems, such as one consisting of two liquids.

This paper presents some experimental observations on the structure of the interface between co-currently flowing oil and water. Laminar-turbulent transitions in both phases were also noted, and probable mechanisms responsible for the first appearance of interfacial waves explored.

Experimental

The arrangement of the experimental set-up is shown in figure 1. Tests were conducted in a rectangular 'Plexiglas' conduit 8·01 in. wide, 1·007 in. high and 37 ft. in overall length. The channel was divided into a 2·5-ft. long liquid entry section, a 12·5-ft. calming section and a 20-ft. test section followed by a 2-ft. discharge section. The sheet metal entry section was designed to bring the two liquids into contact with a minimum of turbulence at the interface. This was followed by a calming section which corresponded to 84 equivalent conduit diameters to allow for the complete development of the flow field prior to entering the test section. The conduit was levelled to within 0·02 in. over its entire length and was free from external vibrations.

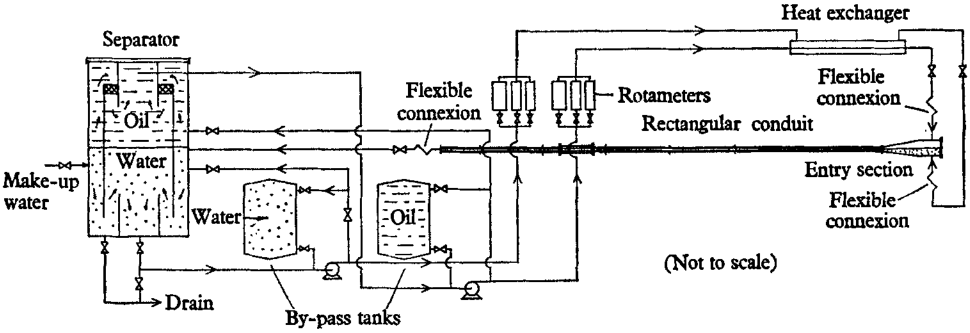

FIGURE 1. Experimental equipment.

Water and a refined mineral oil were chosen for this investigation. The choice of a liquid-liquid system was influenced partly by previous experience with two-phase flow in circular pipes (Charles Govier & Hodgson 1961). Ease of separation of the liquids by gravity and the individual viscosities required to cover significant portions of the laminar and turbulent régimes were among the factors considered in selecting the particular fluids. The properties of these liquids at the conditions of the experiments are given in table 1.

The equipment was capable of being operated at various flow rates corresponding to superficial Reynolds numbers ranging from 46 to 3330 and from 617 to 22,000 for oil and water, respectively.

	Oil*	Water
Specific gravity	0·818	0·997
Viscosity (cP)	4·82	0·905

Oil–water interfacial tension 42 dyn/cm.

* 5 to 1 blend of Mentor 29 and Bayol 35, both supplied by Imperial Oil Limited.

TABLE 1. Physical properties of liquids at 76° F

The superficial Reynolds number of a particular phase is based for the purposes of this discussion on the equivalent diameter of the conduit and the superficial velocity of that phase, defined as the volumetric flow rate of the phase divided by the conduit cross-sectional area.

Actual mean velocity of a phase was also calculated using the flow rate as determined by rotameters, the width of the conduit, and the depth of that phase in the test section. The position of the interface could be determined under flowing conditions when no waves or very small amplitude waves were present by using a pointer gauge or a scale attached to the side of the channel. However, when pronounced waves were present, a stop-flow method was used: oil depth was recorded immediately after the flows of both phases were stopped simultaneously and the interfacial waves had disappeared. Local velocity measurements were restricted to the interface. Very fine aluminium powder was introduced through the top of the conduit and allowed to settle to the oil–water interface where its velocity was measured over a known distance. Wave velocities were obtained similarly by timing the movement of a particular wave over a distance of three feet. Flow régimes for both phases were determined by a dye-injection method. More detailed accounts of the construction and operation of the experimental equipment have been given elsewhere (Charles 1963).

Discussion and results

The experimental observations of the flow régimes and the structure of the interface in this oil–water system are summarized in figure 2.

The effect of a second phase on the observed laminar-turbulent transitions in water and oil are of particular interest. Single phase experiments indicated that the lower critical Reynolds number was 2300 for both fluids, and that the transition to turbulent flow was complete at a superficial Reynolds number of 3500. From figure 2 it is apparent that the presence of a second phase does affect the transition to turbulence in the other phase as indicated by the different critical superficial Reynolds numbers. At very low oil rates, which also correspond to small oil depths, the presence of the lighter and more viscous oil phase seems to have a stabilizing influence on the water layer. An already turbulent water phase, however, induced the transition to turbulence in the oil phase earlier than expected if little or no water was present.

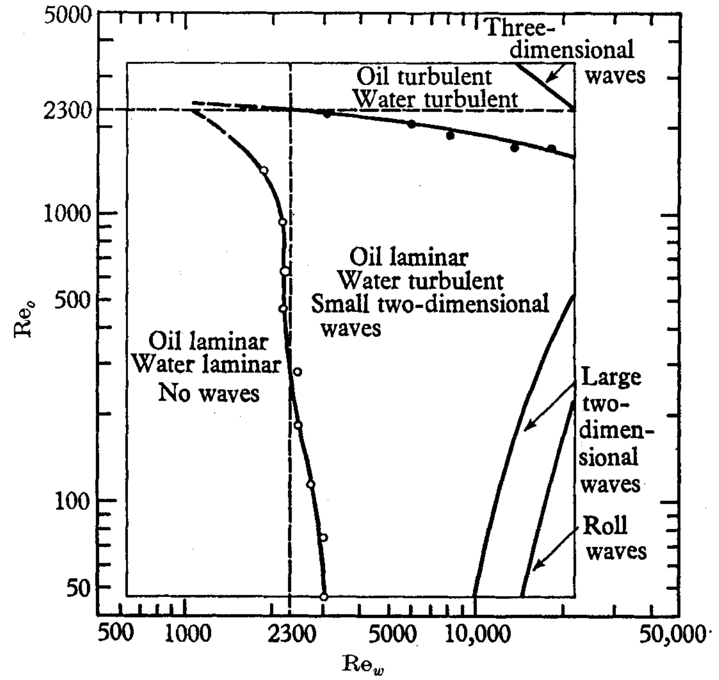

FIGURE 2. Flow régimes and interfacial structure. ○, Laminar-turbulent transition in water phase; ●, laminar-turbulent transition in oil phase; ——, indicates transition in interfacial wave structure.

The region, where both the oil and water phases were in laminar flow, was characterized by a completely smooth interface. Conditions within each phase are illustrated by figure 3A (plate 1), which shows dye traces injected simultaneously into the upper oil and the lower water phases with a superficial oil Reynolds number, Re_0, of 649 and a superficial water Reynolds number, Re_w, of 1225. The discussion of theory and experiment in this two-phase region has been presented in another paper (Charles & Lilleleht 1965). As the water flow rate was slowly increased at a constant oil rate beyond the critical value, first signs of turbulence in the water phase were shown by the appearance of turbulent patches, one of which is shown in figure 3C (plate 1). These turbulent patches travelled downstream and were separated by sections of unbroken dye filaments as in figure 3B indicating intermittent laminar flow. Further increase in the water flow rate caused the turbulent patches to be generated more and more closely together until the transition to turbulence was complete. These observations are similar to those of Rotta (1956) for the single-phase transitional flow in circular pipes. It was also observed that the depth of the water phase momentarily increased by approximately 0·02 to 0·03 in. as a turbulent patch passed the point of observation.

Coincident with the appearance of turbulent patches, the interface was observed to be disturbed by two-dimensional waves, i.e. waves having crests normal to the direction of the flow and several times longer than the wavelength. These waves, some of which are shown in figure 4 (plate 2), had a relatively short wavelength of approximately 0·5 in. and a very small amplitude. There was no doubt that the observed waves were associated with the turbulent patches in the water phase. Between the turbulent patches, where the dye-filament indicated undisturbed flow, the interface was free of waves. Thus the groups of waves travelled at the same velocity as the patches of turbulence. These different conditions are illustrated in figure 3B and 3C for the superficial oil Reynolds number of 649 and the superficial water Reynolds number of 2150. In part B of the figure both phases were apparently in laminar flow and no interfacial waves could be observed. At the same flow rates in part C a patch of turbulence existed in the water phase and associated with this turbulence were waves at the interface. Increased water rates showed the turbulence in the water phase to become continuous with an uninterrupted wave system at the interface as indicated in figure 3D. The dye traces show that the interfacial waves in their early stages of development had little if any effect on the motion in the oil phase. Only as the oil flow itself approached the critical régime did the waves show a more significant effect in the oil phase as demonstrated in figure 3E by an undulating dye filament.

At oil Reynolds numbers below approximately 500 profound changes were noticed in the nature of the two-dimensional waves as the water flow rate was increased. Very large and regular two-dimensional waves appeared. Many of these waves, henceforth referred to as large two-dimensional waves, had crests running across the complete width of the channel and had a wavelength of approximately 3 in. as shown in figure 5 (plate 3). The original short wavelength and short amplitude waves seemed to be superimposed on these larger waves. Further increase in water rate at a constant oil flow rate resulted in additional thinning of the oil layer and in the appearance of roll waves which are illustrated in figure 6 (plate 4). Again small amplitude waves were superimposed upon these interfacial disturbances. Roll waves moved relatively slowly. They had sharp irregular crests and a highly variable wavelength up to approximately 3 in.

Turbulent flow in the oil phase tended to destroy the essentially two-dimensional characteristics of the interfacial waves. As the Reynolds numbers of both phases were increased in the turbulent-turbulent régime, the waves became more and more three-dimensional in character with the crest lengths approximately equal to the wavelength as indicated in figure 7 (plate 5).

Wave velocities were recorded for the three different types of waves: small two-dimensional, large two-dimensional and roll waves. The coincidence of the onset of turbulence in the water phase and the appearance of small two-dimensional waves suggests that these waves were generated and sustained by the turbulent pressure fluctuations as proposed by Phillips (1957). The velocity of these waves is therefore expected to be related to the actual velocity of the water phase. Figure 8 shows that the velocities of the small two-dimensional waves fall indeed very close to the actual average water velocity, thus providing additional support for the applicability of the Phillips resonance mechanism to the generation of waves in this two-liquid system. On the other hand, the velocity of the large two-dimensional and roll waves deviated significantly from the average water velocity. These waves appeared to move much slower than the water phase, and their velocities were also influenced by the oil Reynolds numbers.

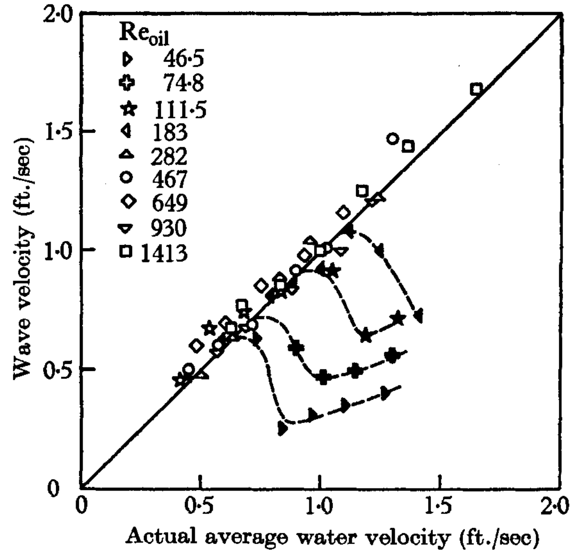

FIGURE 8. Wave velocities with laminar oil and turbulent water. *Note.* Darkened points indicate large two-dimensional waves.

General expressions for the velocity distribution within the two phases in laminar co-current flow in rectangular conduits have been derived theoretically (Charles 1963; Charles & Lilleleht 1965) making the usual assumptions of no slip at the conduit walls and the interface. The predicted and the experimental velocities were compared at the oil–water interface in addition to comparing the actual flow rates of the phases with the integrated velocity profiles. The agreement between the theory and experiment was good in both cases as long as the oil and the water phases remained laminar. As soon as transition to turbulence took place in the water layer, significant deviations in the interfacial velocities became apparent as expected. Figure 9 illustrates these deviations along with showing the agreement that was typical between the laminar-laminar theory and the corresponding data at selected oil flow rates. Here the interfacial velocity w_i in the axial direction equidistant from the two side walls of the conduit has been made dimensionless by the division with V_{rc}, which is the average velocity for the flow of the more viscous oil phase alone in the rectangular conduit under the theoretical two-phase pressure gradient. In this way the pressure gradient is eliminated as a variable and, for given aspect and viscosity ratios, this velocity ratio is predicted by theory to be a unique function of the oil-input volume fraction, which is defined as the volumetric flow rate of the oil divided by the combined flow rates of oil and water. Transition to turbulence in the water phase results in an increased thickness of that phase and a flattening of the velocity profile close to the interface. This in effect decreases the interfacial velocity when the maximum velocity in the laminar-laminar profile had occurred in the oil layer, as was the case with the oil input fractions higher than 0·7 in this particular system. At lower oil input fractions the maximum velocity was in the water phase and the turbulent transition resulted in an increased interfacial velocity. No experimental measurements on the mean velocity profiles within the turbulent water layer have as yet been made in this conduit.

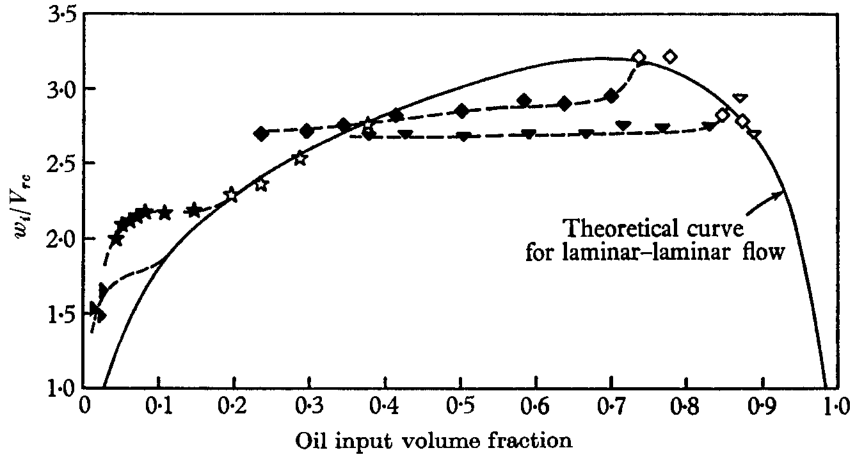

FIGURE 9. Interface velocities.

	Symbols for water phase	
Re_0	Laminar	Turbulent
46·5		▶
111·5	☆	★
649	◇	◆
960	▽	▼

The experimental observations described above indicate that at least two basically different mechanisms may be operative in generating waves at fluid-fluid interfaces. The Phillips' resonance mechanism appears to be responsible for the generation of the first or small two-dimensional waves in the system investigated, whereas the large two-dimensional and roll waves may well appear due to some mean flow instability mechanism. Although no extensive measurements have been made on the mean velocity profiles within either of the phases, there is evidence that the classical laminar distribution does respresent the flow field up to the transition to turbulence.

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